

FERMILAB ENERGY DOUBLER  
MAGNETS-MAGNETOSTATICS

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Summary

Magnetic field computations have been performed for the three-inch circular aperture doubler magnets in the warm iron geometry. Locations for rectangular conductors along circular arcs have been found such that, in the absence of the construction errors, the sextupole and decapole terms have been removed from the dipole. For the quadrupole suitable locations for the conductors have been found that remove the duodecapole term. Field quality, longitudinally integrated fields, construction errors, forces, energy content, and eddy current heating under cycled conditions will be discussed for a 45 kG dipole and a 20 kG/inch quadrupole.

Design Considerations

The possibilities inherent in circular iron shields, elliptical iron shields, pancake coils, offset circular shell coils, and circular shell coils have been examined using complex variable methods.<sup>1</sup> Both the field in the transverse section and the longitudinally integrated field are calculated. For each geometry a search mode is employed to improve any one of several parameters specifying initial conductor locations. Thus, for circular shells, the radial position, azimuthal position, or azimuthal space between keystoned conductors may be adjusted. For pancake coils, the horizontal position or horizontal space between rectangular conductors may be varied. The end result of the search is a set of conductor locations that minimizes the energy content within the reference radius of all multipoles except the lowest. Although the search procedure may be incorporated into the longitudinally integrated fields, this was not done because, for relatively long magnets, the end effects are small and a few runs suffice to obtain the desired quality.

Additional calculations provide the field distribution and net flux entering the iron shield, eddy currents induced in various elements, and the electromagnetic force distribution in the conductors. Thus one may estimate field modifications induced by iron saturation, the iron cross section necessary to reduce return flux saturation, and the power loss in the bore tube liner, cryostat walls, and heat shield. From the force calculations realistic estimates are made of the banding tension necessary to restrict conductor movements and the spring constant<sup>2</sup> for the displacement of the coil package relative to the iron.

An iron shield in the shape of an upright ellipse has a reduced saturation effect on the field distribution and yields a coil

arrangement for high quality fields that reflects the confocal nature of the coordinate system. Hence the usable aperture has even higher eccentricity than the shield although minimal net flux results. A horizontal aperture sufficiently small to provide an economic advantage over the circular case requires vertical injection and extraction. This imposes a considerable constraint on the elliptical shield which, therefore, was abandoned in favor of the circular shield. Having chosen a circular shield, the concentric nature of the coordinate system dictates that for high quality fields the conductor arrangement be circular. Field quality demands from orbit considerations, extraction requirements, and achievable construction tolerances set the aperture diameter at about three inches.

Considerations that lead to a choice of inner iron radius are as follow. In a typical cold iron design the shield is used directly to hold the coil package in place. Saturation effects on the magnetic field must be counterbalanced with additional correction windings. This design, however, makes maximum use of the iron in producing field. If, on the other hand, it is desired to have the dipoles and quadrupoles track with an accuracy sufficient to permit a single excitation current throughout the magnet system, then, for the coil package that yields a three inch aperture, the inner iron radius must be about four inches. For this radius there is sufficient space between the outside of the coil package and the iron to insert a thermally insulating support structure. Thus, the iron may remain at room temperature. In this warm iron design, although there is no significant saturation effect, more ampere-turns must be provided to offset the diminished utilization of the iron. We have opted for the warm iron design with the attendant possibility of simplifying the power distribution system.

For the most economical use of superconductor one may tailor the conductor size depending on its location in the magnetic field. Thus, in the dipole, two grades of multistrand cable were used. For the inner two shells a seven-strand cable of nominal size .150 in by .075 in was chosen<sup>3</sup> for which the effective JB product is 80% of short sample.<sup>4</sup> In the outer two shells an 11-strand cable of nominal size .150 in by .050 in was used for which the effective JB product is 70% of short sample.

Having chosen the space allowed in the dipole for conductors, banding, cryostats, and supports, these general space allocations were incorporated into the quadrupole for maximum simplicity. These conditions and the desire to obtain the highest gradient possible in order to minimize longitudinal space allocation dictate an ungraded conductor design for the quadrupole. Thus the 11-strand conductor

was chosen which operates with a JB product at 63% of short sample.<sup>4</sup>

The length of the iron shield relative to the coil ends must be chosen. Calculations using many segments of linear current elements have been made<sup>3</sup> and indicate that for dipoles, in the absence of the iron shield, a field enhancement of some 20% is expected in the end region. Since the field enhancement by the iron in the transverse section is 18% it is desirable to terminate the iron somewhat before the conductors are turned around. In this manner there will be no significant field enhancement in the ends. For the quadrupole, since the maximum field is much less than 45 kG, the iron shield may be carried out over the coil ends.

Finally, it is to be noted that in construction both the dipoles and the quadrupoles the construction tolerance<sup>5</sup> on the location of conductor shell radii and azimuthal position of the shell is  $\pm 0.002$  in.

Tables 1 - 8 are self explanatory. Table 9 refers to the longitudinally integrated fields in which circular turn-around ends are used, the turn centers being separated by the length indicated. The entries T(N), S(N), and R(N) refer to coefficients in a multipole expansion of the longitudinally integrated magnetic field. Successive terms give  $\Delta B$  at the reference radius for the dipole, sextupole, decapole, etc. The contribution due to the currents with no shield is T(N), the contribution from the iron shield is S(N), and R(N) is the ratio T(N) + S(N) divided by T(1) + S(1). Table 10 is a similar calculation in which the contribution due to the ends is omitted and the length set equal to one inch. The median plane field in the transverse section is given by BT. Columns BA, BS, and BN give respectively the contribution in the absence of the shield, the contribution due to the shield, and the total field normalized to unity at the origin. The entry DELR(N) is an estimate<sup>6</sup> of the magnitude of the change in R(N) induced by saturation effects in the iron shield. Tables 11 - 12 provide similar information relative to the quadrupole, T(N) etc. now stepping through quadrupole, duodecapole, etc.

#### References

1. W.W. Lee and S.C. Snowden, IEEE Trans. on Nucl. Sci., NS-20, 726 (1973)
2. K. Halbach, Nucl. Instr. and Methods, 78, 185 (1970)
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4. P. Price and R. Yamada, Fermilab Technical Note TM-538, Nov. 23, 1974
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6. M.A. Green, Kernforschungszentrum Karlsruhe External Report 3/71-7, June 1972

Table 1. Bending Magnets  
Performance Parameters

Field Strength	45 kG
Effective Field Length	240 in
Good Field Width	2.0 in
Field Quality ( $\Delta B/B$ at 1 in Rad.)	$\pm 0.05\%$

Table 2. Design Data for  
Bending Magnet

Conductor Current (45 kG)	2345 A
Conductor Size (no insulation)	
Inner 2 shells	
(7-strand)	.152 in by (.075/.0636) in
Outer 2 shells	
(11-strand)	.152 in by (.050/.0432) in
Effective Current Density	
(7-strand)	215 kA/in <sup>2</sup>
(11-strand)	320 kA/in <sup>2</sup>
Total Number of Turns	228
Insulation Thickness (spiral wrap)	.004 in
Inner Bore Tube Radius (304 SS)	1.125 in
Inner Bore Tube Wall Thickness	.050 in
Inner Cryostat Radius (304 SS)	2.50 in
Inner Cryostat Wall Thickness	.018 in
Outer Cryostat Radius (304 SS)	2.625 in
Outer Cryostat Wall Thickness	.018 in
Lamination Inner Radius (mild steel)	4.00 in
Lamination Thickness	.0625 in
Outside Dimension of Iron	16 in by 10 in
Total Length of Iron	234 in

Table 3. Stored Energy and Losses  
in Bending Magnet

Peak Stored Energy	.54 MJ
Inductance	.18 Hy
Repetition Period	60 sec
Eddy Current Losses	
Bore Tube	.13 W
Conductor Matrix	1.8 W
Inner Cryostat	.22 W
Outer Cryostat	.22 W
Heat Shield (200K)	.2 W
Lamination (warm)	neg.
Hysteresis Losses	
Superconductor	3.3 W
Lamination (warm)	3.5 W

Table 4. Forces and Critical  
Fields in Bending Magnet

Central Field		45 kG
Maximum Field in Conductor		
Inner 2 shells (7-strand)		47 kG
Outer 2 shells (11-strand)		38 kG
Effective Radius of		
Conductor shells		1.896 in
Traction at Effective Radius		
Angle	x-Traction	y-Traction
(Deg)	(lb/in <sup>2</sup> )	(lb/in <sup>2</sup> )
0	699	0
40	1521	-1174
50	1623	-985
90	0	0
Displacement Force		
(x-displ. = .010 in)		9.91b/in
(y-displ. = .010 in)		9.91b/in

**Table 5. Focusing Magnet  
Performance Parameters**

Gradient Strength	20.7 kG/in
Effective Gradient Length	62 in
Good Field Width	2.0 in
Gradient Quality ( $\Delta B/xB'$ at 1 in Rad.)	$\pm 1.2\%$

**Table 7. Stored Energy and Losses  
in Focusing Magnet**

Peak Stored Energy	51 kJ
Inductance	.013 Hy
Repetition Period	60 sec
Eddy Current Losses	
Bore Tube	.002 W
Conductor Matrix	.20 W
Inner Cryostat	.004 W
Outer Cryostat	.004 W
Heat Shield (20°K)	.003 W
Lamination (warm)	neg.
Hysteresis Losses	
Superconductor	.56 W
Lamination (warm)	.25 W

**Table 6. Design Data for  
Focusing Magnet**

Conductor Current (20.7 kG/in)	2345 A
Conductor Size (no insulation)	.152 in by (.050/.0432) in
Effective Current Density	320 kA/in <sup>2</sup>
Turns per Pole	51
Insulation Thickness (spiral wrap)	.004 in
Inner Bore Tube Radius (304 SS)	1.125 in
Inner Bore Tube Wall Thickness	.050 in
Inner Cryostat Radius (304 SS)	2.50 in
Inner Cryostat Wall Thickness	.018 in
Outer Cryostat Radius (304 SS)	2.625 in
Outer Cryostat Wall Thickness	.018 in
Lamination Inner Radius (mild steel)	4.00 in
Lamination Thickness	.0625 in
Outside Dimension of Iron	10.0 in Dia.
Total Length of Iron	66 in

**Table 8. Forces and Critical Fields  
in Focusing Magnet**

Central Gradient	20.7 kG/in	
Maximum Field in Superconductor	31 kG	
Effective Radius of Superconductor	1.838 in	
Traction at Effective Radius		
Angle (Deg)	x-Traction (lb/in <sup>2</sup> )	y-Traction (lb/in <sup>2</sup> )
0	136	0
20	612	-1448
25	714	-1399
45	0	0
65	-1399	714
70	-1448	612
90	0	136
Displacement Force		
(x-displ. = .010 in)		2.7lb/in
(y-displ. = .010 in)		2.7lb/in

**Table 9. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES GRADED DOUBLER DIPOLE**

ORDER OF POLE	=	1	CALCULATIONAL MODE	=	1	NUMBER OF LAYERS	=	4			
HIGHEST MULTIPOLE ORDER	=	19	CONDUCTOR CURRENT(A)	=	2345.0000	REFERENCE RADIUS(IN)	=	1.0000			
INNER IRON RADIUS(IN)	=	4.0300	SIMPSON'S RULE INTERVAL(DEG)	=	1.0000	HORIZONTAL INCREMENT(IN)	=	.1000			
INSULATION THICKNESS(IN)	=	.0040									
LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)			
1	29.03	215.531	.1267	81.7456	.00045	1.5000	1.6650	234.0000			
2	27.87	215.531	.1134	68.1899	.00027	1.6800	1.8450	236.0000			
3	30.00	320.522	.0999	45.8796	-.00062	1.9450	2.1100	239.0000			
4	28.00	320.522	.0908	39.4065	-.00052	2.1250	2.2900	240.0000			
MULTIPOLE COEFFICIENTS											
T(IN)	=	8.818E+03	-9.544E+00	-2.327E+00	-1.054E+01	-1.577E-01	5.659E-01	-2.392E-01	7.678E-02	-5.920E-02	2.781E-02
S(IN)	=	2.012E+03	6.692E+00	-5.665E-02	-1.269E-03	2.295E-06	2.709E-07	5.746E-10	-6.172E-11	-6.610E-13	9.761E-15
R(IN)	=	1.600E+00	-2.633E-04	-2.201E-04	-9.733E-04	-1.456E-05	5.225E-05	-2.209E-05	7.089E-06	-5.466E-06	2.563E-06
X(IN)	9T(KG-IN)		8A(KG-IN)		8S(KG-IN)		8N				
.00000	10833.13097		8918.04659		2012.08442		1.00000				
.10000	10833.10220		8817.35196		2012.15134		1.00000				
.20000	10830.31240		8817.66038		2012.35202		.99999				
.30000	10824.84728		8817.16192		2012.68627		.99997				
.40000	10829.57639		8816.41666		2013.15373		.99995				
.50000	10829.16414		8815.35321		2013.75393		.99991				
.60000	10823.30377		8813.81753		2014.43624		.99983				
.70000	10826.92438		8811.57499		2015.34988		.99970				
.80000	10824.58596		8808.24202		2016.34394		.99949				
.90000	10820.72729		8803.25997		2017.46732		.99913				
1.00000	10814.56807		8795.84330		2018.71877		.99856				
1.10000	10805.66826		8784.97138		2020.03648		.99769				
1.20000	10790.92907		8769.32903		2021.60104		.99638				
1.30000	10773.77413		8747.54767		2023.22646		.99452				
1.40000	10744.60066		8719.02652		2024.97414		.99205				
1.50000	10713.75622		8686.91535		2026.84087		.98925				

Table 10. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES GRADED DOUBLER DIPOLE

ORDER OF POLE	=	1	CALCULATIONAL MODE	=	0	NUMBER OF LAYERS	=	4
HIGHEST MULTIPOLE ORDER	=	14	CONDUCTOR CURRENT(A)	=	2345.0000	REFERENCE RADIUS(IN)	=	1.0000
INNER IRON RADIUS(IN)	=	4.0000	SIMPSONS RULE INTERVAL(DEG)	=	1.0000	HORIZONTAL INCREMENT(IN)	=	1.0000
INSULATION THICKNESS(IN)	=	.0040						
LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	29.00	320.522	1.4482	34.1045	-0.0164	1.5000	1.6650	1.0000
2	27.00	320.522	1.3003	25.6932	-0.0108	1.6800	1.8450	1.0000
3	30.00	320.522	1.1304	18.0238	-0.0082	1.9450	2.1100	1.0000
4	28.00	320.522	1.0382	13.7738	-0.0052	2.1250	2.2900	1.0000
MULTIPOLE COEFFICIENTS								
T(IN)	=	3.664E-01	-2.861E-02	1.709E-04	-4.335E-02	-4.386E-04	2.444E-03	-1.031E-03
S(IN)	=	8.336E-00	2.818E-02	-2.287E-04	-5.220E-02	9.464E-03	1.133E-09	2.939E-12
R(IN)	=	1.000E-00	-9.641E-06	-1.244E-08	-9.639E-04	5.346E-05	-2.226E-05	7.479E-06
DEL R(IN)	=	7.574E-04	1.579E-05	5.417E-07	2.642E-08	1.284E-09	6.567E-11	3.475E-12
X(IN)	=	4.499039	36.64438	8.33597	1.00000			
GT(KG-IN/IN)	=	4.499039	36.64438	8.33597	1.00000			
GA(KG-IN/IN)	=	4.499039	36.64438	8.33597	1.00000			
GS(KG-IN/IN)	=	4.499039	36.64438	8.33597	1.00000			
GN	=	4.499039	36.64438	8.33597	1.00000			
MAX. FIELD ON IRON(KG)								
		=	15.644	IRON PERMEABILITY AT BMAXFE=		233.6375	FLUX IN IRON(KG-IN)	
							=	
							67.7712	

Table 11. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES UNGRADED DOUBLER QUADRUPOLE

ORDER OF POLE	=	2	CALCULATIONAL MODE	=	1	NUMBER OF LAYERS	=	4
HIGHEST MULTIPOLE ORDER	=	14	CONDUCTOR CURRENT(A)	=	2345.0000	REFERENCE RADIUS(IN)	=	1.0000
INNER IRON RADIUS(IN)	=	4.0000	SIMPSONS RULE INTERVAL(DEG)	=	1.0000	HORIZONTAL INCREMENT(IN)	=	1.0000
INSULATION THICKNESS(IN)	=	.0040						
LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	17.00	320.522	1.4482	34.1045	-0.0164	1.5000	1.6650	59.0000
2	14.00	320.522	1.3003	25.6932	-0.0108	1.6800	1.8450	63.0000
3	11.00	320.522	1.1304	18.0238	-0.0082	1.9450	2.1100	61.0000
4	9.00	320.522	1.0382	13.7738	-0.0052	2.1250	2.2900	62.0000
MULTIPOLE COEFFICIENTS								
T(IN)	=	1.223E-03	-5.579E-01	-2.927E-06	1.270E-01	-5.742E-02		
S(IN)	=	5.598E-01	5.764E-03	2.541E-07	-2.162E-16	-1.063E-13		
R(IN)	=	1.000E-00	-4.313E-04	-2.258E-03	9.931E-05	-4.488E-05		
X(IN)	=	1279.25113	1221.35889	55.89924	1.00000			
GT(KG-IN/IN)	=	1279.25113	1221.35889	55.89924	1.00000			
GA(KG-IN/IN)	=	1279.25113	1221.35889	55.89924	1.00000			
GS(KG-IN/IN)	=	1279.25113	1221.35889	55.89924	1.00000			
GN	=	1279.25113	1221.35889	55.89924	1.00000			
MAX. FIELD ON IRON(KG)								
		=	6.9576	IRON PERMEABILITY AT BMAXFE=		667.6402	FLUX IN IRON(KG-IN)	
							=	
							14.4223	

Table 12. INTEGRATED MULTIPOLE STRUCTURE OF C-SERIES UNGRADED DOUBLER QUADRUPOLE

ORDER OF POLE	=	2	CALCULATIONAL MODE	=	0	NUMBER OF LAYERS	=	4
HIGHEST MULTIPOLE ORDER	=	10	CONDUCTOR CURRENT(A)	=	2345.0000	REFERENCE RADIUS(IN)	=	1.0000
INNER IRON RADIUS(IN)	=	4.0000	SIMPSONS RULE INTERVAL(DEG)	=	1.0000	HORIZONTAL INCREMENT(IN)	=	1.0000
INSULATION THICKNESS(IN)	=	.0040						
LAYER	TURNS	CURDEN (KA/IN/IN)	THETAS (DEG)	THETAF (DEG)	SPACER (IN)	RINNER (IN)	ROUTER (IN)	LENGTH (IN)
1	17.00	320.522	1.4482	34.1045	-0.0164	1.5000	1.6650	1.0000
2	14.00	320.522	1.3003	25.6932	-0.0108	1.6800	1.8450	1.0000
3	11.00	320.522	1.1304	18.0238	-0.0082	1.9450	2.1100	1.0000
4	9.00	320.522	1.0382	13.7738	-0.0052	2.1250	2.2900	1.0000
MULTIPOLE COEFFICIENTS								
T(IN)	=	1.979E-01	4.129E-06	-4.792E-02	2.240E-03	-9.584E-04		
S(IN)	=	8.933E-01	9.374E-05	4.487E-09	-3.348E-12	-1.677E-15		
R(IN)	=	1.000E-00	4.734E-06	-2.317E-03	1.083E-04	-4.634E-05		
DEL R(IN)	=	6.170E-05	6.044E-08	1.585E-10	5.260E-13	1.598E-15		
X(IN)	=	20.63322	19.78998	.89335	1.00000			
GT(KG-IN/IN)	=	20.63322	19.78998	.89335	1.00000			
GA(KG-IN/IN)	=	20.63322	19.78998	.89335	1.00000			
GS(KG-IN/IN)	=	20.63322	19.78998	.89335	1.00000			
GN	=	20.63322	19.78998	.89335	1.00000			
MAX. FIELD ON IRON(KG)								
		=	6.9576	IRON PERMEABILITY AT BMAXFE=		667.6402	FLUX IN IRON(KG-IN)	
							=	
							14.4223	